Required for students who earned less than an A on the last test.

Pre Calculus Honors
Unit 5 Trigonometry Graphs Test Prep - DUE _____

1. To prepare for your test **add content to your previous personal study guide.** Use the index below to pull out important topics from Unit 5

Prior to Spring Break

5.1

A. Parent Graphs (Sine, Cosine), key vocabulary/terms

5.2

A. Period, Amplitude, and Horizontal Midway Line Transformations (Basic)
B. Writing equations from graphs - (including period, amp, vertical shift, and reflection over x-axis)
C. Period, Amplitude, and Vertical Shift - Graphing and Writing from Graphs (all values a, b, c)

5.3

A. Phase Shift (basic)
B. Phase and Period shift
C. Writing equations from graph involving phase and/or period shift with or without starting point

This Week

5.4 - Application Problems

A. General trends, reading graphs, writing graphs to match real-world scenarios
B. Time, Tidal Flows, and Ferris Wheels

2 . **Complete at least fifteen problems** from the bank on the back of this study guide. Focus on the areas you need to practice, but make sure you look over all types of problems/topics. *The answer key is attached. I will be checking your work for the fifteen problems.* This guide is meant to be a general, *basic* review of the topics covered in Unit 5

3. **Review** your quizzes, homework, classwork practice, and warm-ups. Practice specifically with the problems you missed!

Your study guide (3 points) and the practice problems (2 points) are homework grades.

Your test is on Tuesday, April 12. You are welcome to attend a tutorial or stop by during lunch (any day but Tuesday) with specific questions or areas to review. Please let Mrs. Pike know in advance that you are coming!

PC 12 LG 7 Worksheet (Trig Graphs)

1. If $y = -5\text{Sin}\left(2x + \frac{\pi}{8}\right) - 7$, find the:

a) amplitude
b) vertical displacement
c) period
d. phase shift
e. max value of y
f) min value of y

2. If $y = 3\text{Cos}(6\pi x - 12) + 14$, find the:

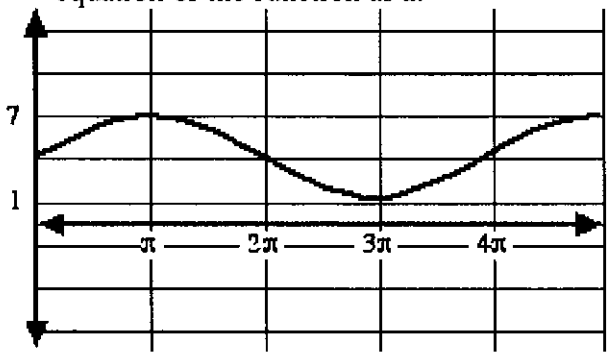
a) amplitude
b) vertical displacement
c) period
d. phase shift
e. max value of y
f) min value of y

3. If $y = -4\text{Cos}(2x - 8) - 10$, find the:

a) domain of the function.

b) range of the function.

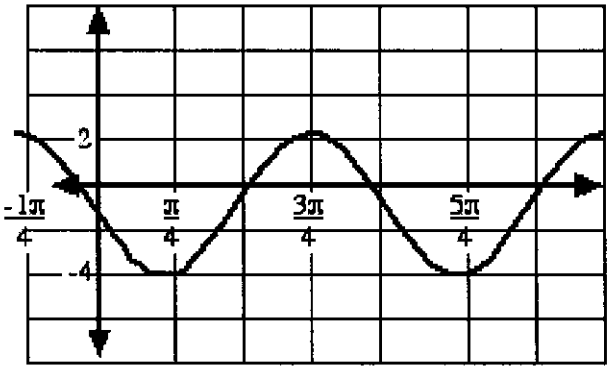
4. Given the sinusoidal graph below, write the equation of the function as a:



a) Sine function

b) Cosine function

5. Given the sinusoidal graph below, write the equation of the function as a:



a) Sine function

b) Cosine function

6. a) Find the maximum value of $f(x) = a\text{Sin}x + d$ where $a > 0, d > 0$.

b) Determine the period of $y = 8\text{Cos}\frac{2\pi}{15}x + 8$.

c) Determine the range of $y = 4\text{Cos}x - 2$.

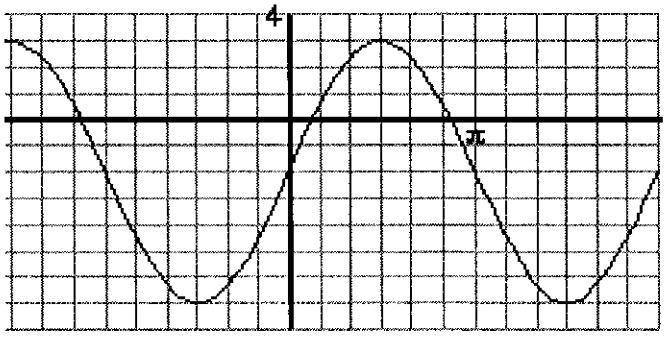
d) Determine the range of $y = -2\text{Sin}3x + 4$.

e) Determine the period of $f(x) = \frac{-1}{2}\text{Sin}\frac{x}{3}$.

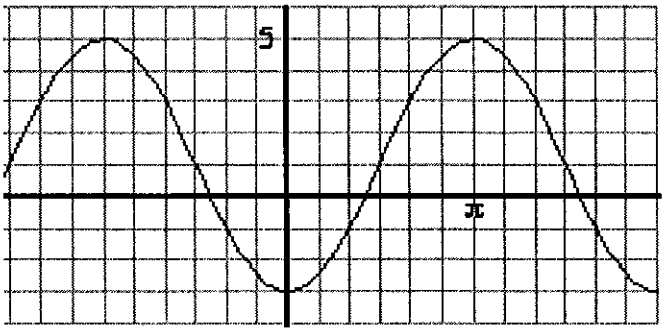
f) Find the range of $f(x) = b\text{Cos}ax - 2b$ where $a > 0, b > 0$.

7. Given the graphs below, determine an equation of the function. Use your graphing calculator to check.

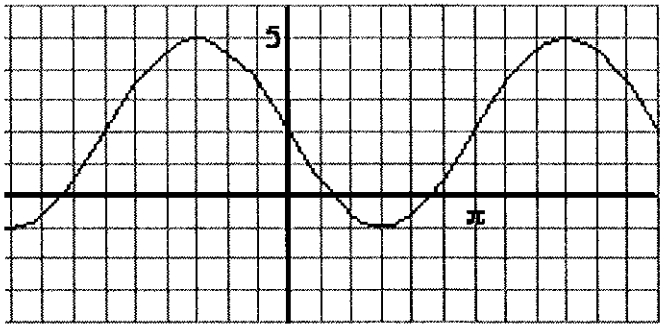
a)



b)

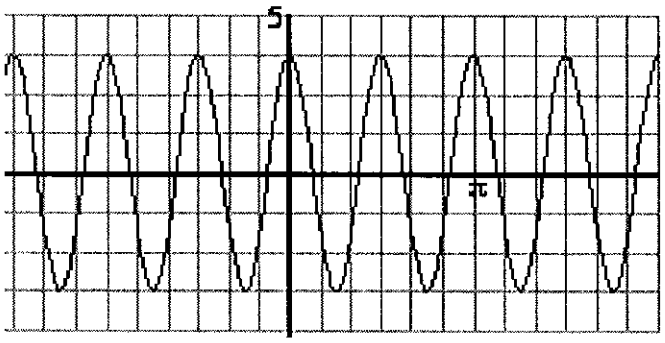


c)

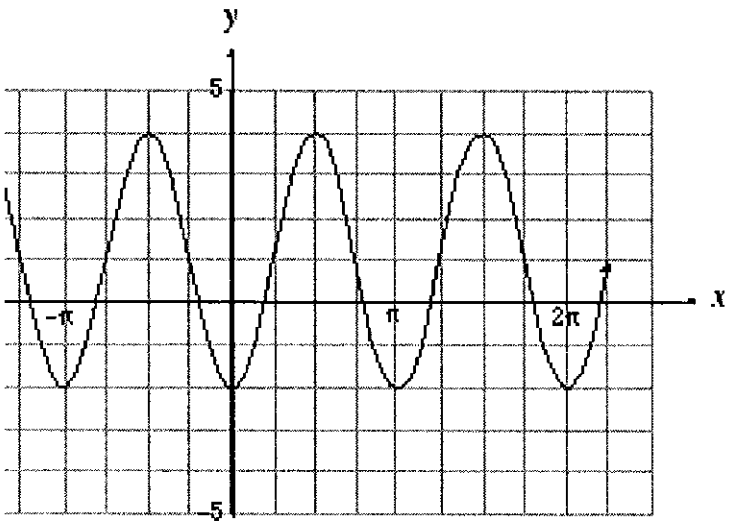


PC 12 LG 7 Worksheet (Trig Graphs)

d)



8. The graph below is a function that can be written in the form: $y = a\text{Sin } b(x - c) + d$. Determine the values of a, b, c, d. Use your graphing calculator to verify your result.



9. At a seaport, the depth of the water h in metres at time $t = 1$ hour during a certain day is given by this formula

$$h = 2.4\text{Cos}\frac{2\pi}{12.4}(t - 5) + 4.2$$

Assume that when $t = 0$, it is midnight. Use your graphing calculator to determine how long after midnight (the first two times) that the water is 5 m deep.

Answer Key

1. a. 5
b. -7
c. π
d. $\frac{\pi}{16}$ left
e. -2
f. -12

2. a. 3
b. 14
c. $\frac{1}{3}$
d. $\frac{2}{\pi}$ right
e. 17
f. 11

3. a) All real #'s
b) $-14 \le y \le -6$

4. a. $y = \sin\frac{1}{2}(x - 0) + 2$

b. $y = \cos\frac{1}{2}(x - \pi) + 2$

5. a. $y = 3\text{Sin } 2(x - \frac{\pi}{2}) - 1$

b. $y = 3\text{Cos } 2(x + \frac{\pi}{4}) - 1$

6. a. $a + d$
b. 15
c. $-6 \le y \le 2$
d. $2 \le y \le 6$
e. 6π
f. $-3b \le y \le -b$

note: *there are many different possibilities for the answers to questions 7 and 8.*

7. a) $y = 5\text{Sin } x - 2$
b) $y = -4\text{Cos } x + 1$
c) $y = -3\text{Sin } x + 2$
d) $y = 3\text{Cos } 4x$

8. $a = 3, b = 2, c = \frac{\pi}{4}, d = 1$

or

$a = -3, b = 2, c = \frac{-\pi}{4}, d = 1$

9. 2.57 hrs, 7.43 hrs

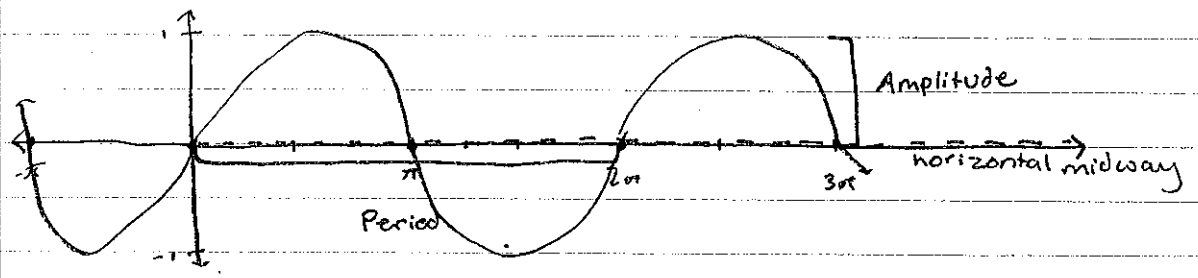
Benjamin Maggie 5.2A

5.2 A - Period, Amplitude and Horizontal Midway Transformations

Period: The horizontal distance of one wave

Amplitude: The height of the wave, measured from the horizontal midway

Horizontal Midway Line: The line which cuts the range of a sine/cosine graph in half

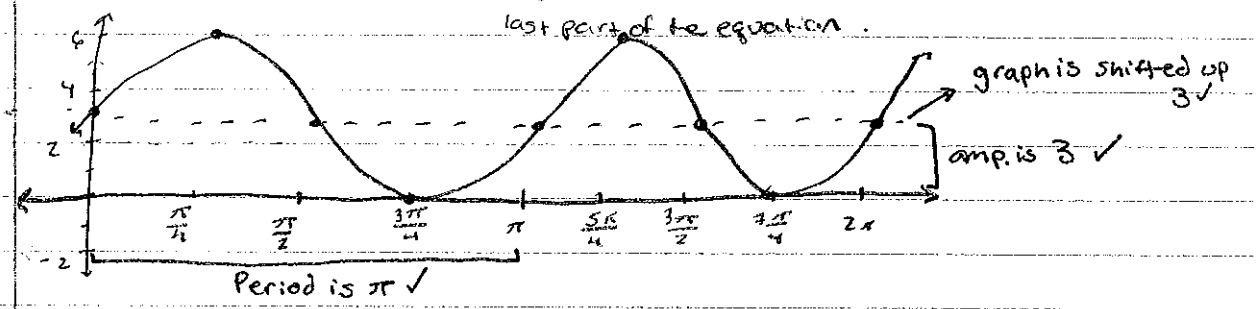


ex. $y = 3\sin(2x) + 3$

↳ amplitude is 3 → will appear as a coefficient to either sin or cos

↳ period is π → to find the period divide 2π by the coefficient of x. In this case, $2\pi/2$ is π, so the period is π

↳ horizontal midway line is at positive 3 → look for this as the last part of the equation.



Tips

1. Draw in the horizontal midway line
2. Write out...
   - Amp:
   - Period:
   - Vertical Shift:
   - Sin/Cos:
   - Phase shift:

   Before you graph.

To Remember

1. PERIOD = $\frac{2\pi}{\text{coefficient of } x}$
   ↳ the greater the coefficient, the smaller the period
2. Horizontal Midway Line is the equivalent of the x-axis from the parent function's graph. The amplitude is up or down from the horizontal midway line.

Isaiah J.
Eliot "thesnake" Bilshi

5.2B

⚠ watch out! dont confuse amplitude with with V.S! Your vertical shift is what determines where your horizontal midline is, not your amplitude

Key Vocab:

Amplitude - Distance from center of wave to crest of wave. (Half the height of wave)

Trick: To find period, divide $\frac{2\pi}{b}$, then you're ready to have fun fun fun fun fun fun

Period - Section of graph that is repeated over and over again.

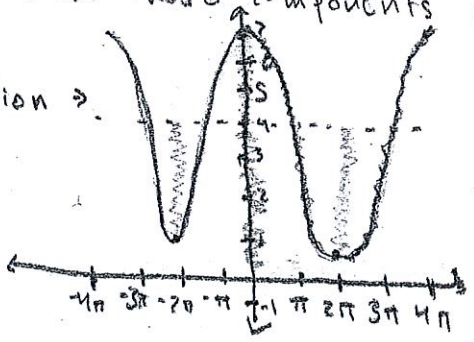
Vert. Shift - Entire graph is moved up or down y-axis

ex: $y = 3\cos(.5x) + 4$

step 1: Identify Key components of equation → $y = \underbrace{3}_{\text{amplitude}}\cos(\underbrace{.5}_{\text{Period}}x) + \underbrace{4}_{\text{vertical shift}}$

step 2: Evaluate what those components mean → graph is cosine

step 3: Graph equation →



$y = 3\cos(.5x) + 4$

- 3: crest's distance from center is 3 units
- .5: Period will be $\frac{2\pi}{.5} = 4\pi$
- 4: graph is moved up 4 units

Tricks and Tips:

for equation: $y = a\sin(bx) + c$

Tip: when evaluating your equation check to see if your a is negative, if so, flip graph over x-axis.

Tip: when getting an equation from graph, simply do the 3 steps in reverse! First examine your graphed equation, then identify what the amp. per. and vert. shift are, then write those in the equation!

Notes (5.3) A

Phase shift (basic): Moves the graph left or right on the x-axis

$-$ (− left, + right)

$$y = a\sin(bx - \underbrace{y}_{\text{Phase shift}}) + c$$

Phase and Period Shift!

vocab:

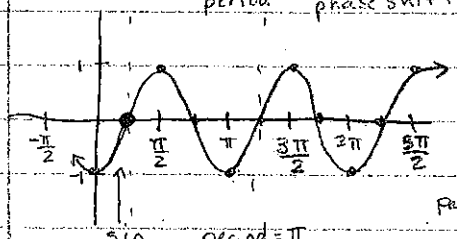
- Phase shift: the horizontal movement of an entire graph
- Period: the length of one full wave on the x-axis (typically)

examples:

$y = \sin(2x - 60°)$ given

$y = \sin(2(x - 30°))$ factor to isolate x

period, phase shift



sin

period = π

$\frac{\pi}{4}$ → phase shift

$y = \sin(\_(x - \_))$

$y = \sin(2(x - \frac{\pi}{4}))$ phase shift

period $y = \sin(2x - \frac{\pi}{2})$

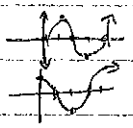
→ can be written in terms of cosine as well...

$y = \cos(2(x - \frac{\pi}{2}))$

remember $\frac{2\pi}{b}$ = period

tips and things to remember

- period before phase. figure out your period first to avoid later confusion.
- double check the sign of your phase shift!!!
- waves often work in fives. {for sin: center, max, center, min, center / for cos: max, center, min, center, max} figure out where these points hit and you've basically got your graph.



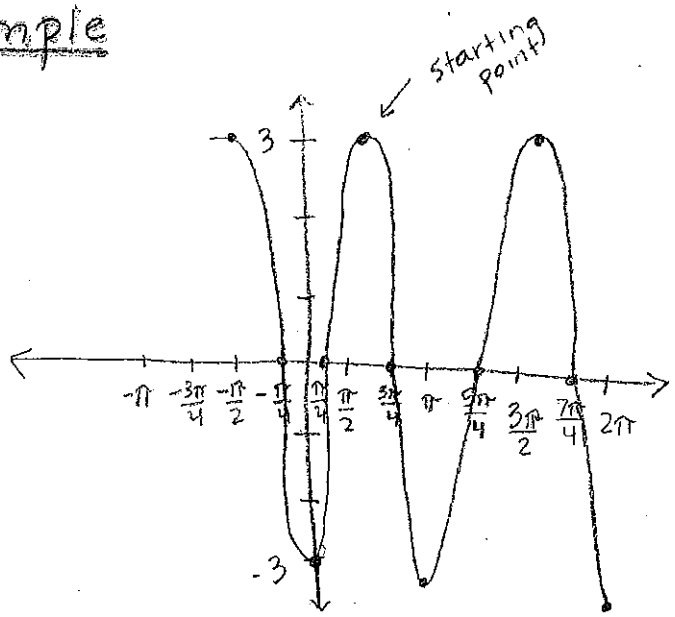
- always double check: you might've missed a negative sign.

- phase shift = horizontal shift

Steps

1. Determine if the function is Sine or Cosine
2. Find the vertical shift
3. Find the change in amplitude
4. Determine the change in period
5. Determine the phase shift

Example



1. Parent Function is Cosine
2. No vertical shift
3. Amplitude is 3
4. Period is π
5. Phase shift is $\frac{\pi}{2}$ to the right

$$f(x) = 3\cos\left(2\left(x - \frac{\pi}{2}\right)\right)$$

↑ amplitude; ↑ period change; ↑ phase shift

Things to Remember

- if graph is shifted to the left π then equation is $(x+\pi)$. If shifted to the right π then equation is $(x-\pi)$.
- When writing the equation, write completely factored to avoid changing the phase shift

ex. $3\cos(2(x-\frac{\pi}{2}))$ ✓

$3\cos(2x-\pi)$ ✓

$3\cos(2x-\frac{\pi}{2})$ ✗

↑ did not distribute 2 to everything

★ If no starting point is given, graph can be written in both Sine and Cosine

5.~~B~~A

Practice Problems

Graph the equation:

1) $3\sin(x)$

2) $\cos\left(\frac{1}{2}x\right)$

3) $\cos(x)+2$

4) $2\sin(2x)-3$

5) $\frac{1}{2}\cos\left(\frac{1}{2}x\right)+1$

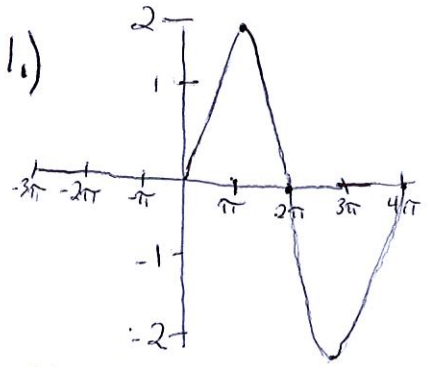
5.2

5.2 B

Practice Problems:

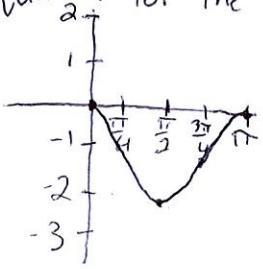
For problems 1-3 create equation for the graphs shown

1.)



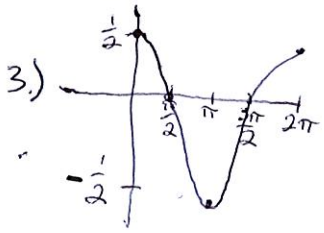
y=

2.)



y=

3.)



y=

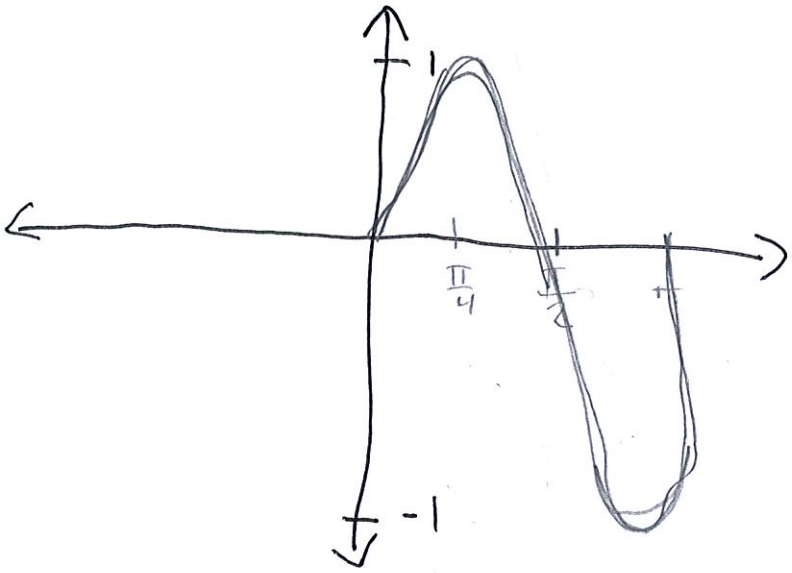
For problems 4&5 sketch graph for the equations shown

4.) $y = 4|\sin(2x)| + 1$

5.) $y = \cos\left(\frac{1}{2}x\right) + 3$

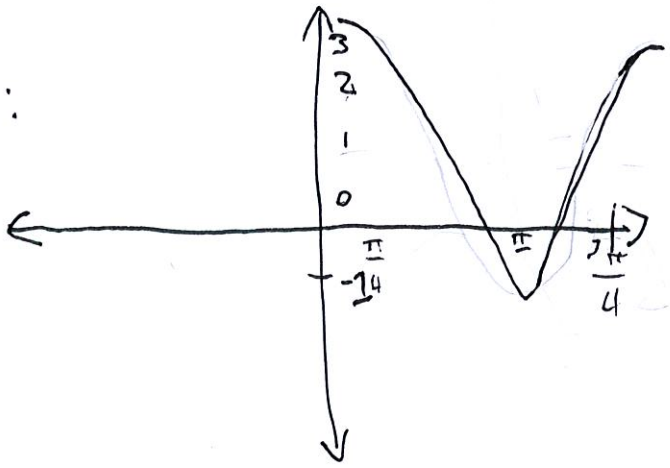
#1: $2\sin\left(\frac{1}{2}x\right)+2$

#2:



#3: $3\cos(x)-1$

#4:



#5: $2\cos(2x)+2$

#1: $\sin(x+\pi)$

#2: $\cos\left(x-\frac{\pi}{2}\right)+1$

#3: $2\sin(x-\pi)-1$

#4: $\frac{1}{2}\sin(x+\pi)-2$

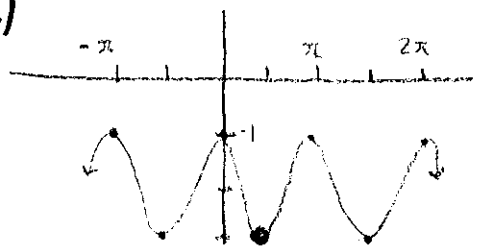
#5: $2\sin\left(x-\frac{\pi}{4}\right)-1$

Long

5.3c

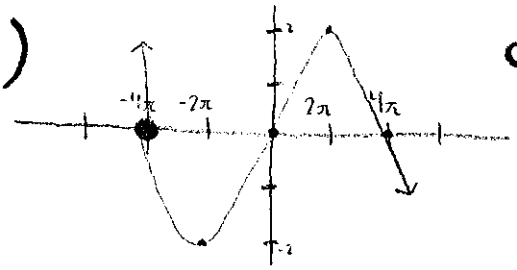
1. Identify each graph as sine or cosine and find each of the listed characteristics based on the given starting point

a)



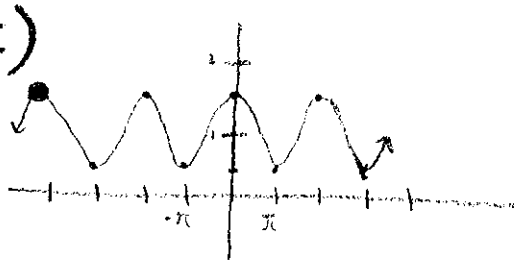
Sine or cosine
phase shift:
period:

b)



Sine or cosine
period:
amplitude:
phase shift:

c)



Sine or cosine
phase shift:
amplitude:

2. Match each equation with the corresponding characteristics.

a. $3\sin(x-2)+1$

b. $\frac{1}{2}\cos(2(x+2))-4$

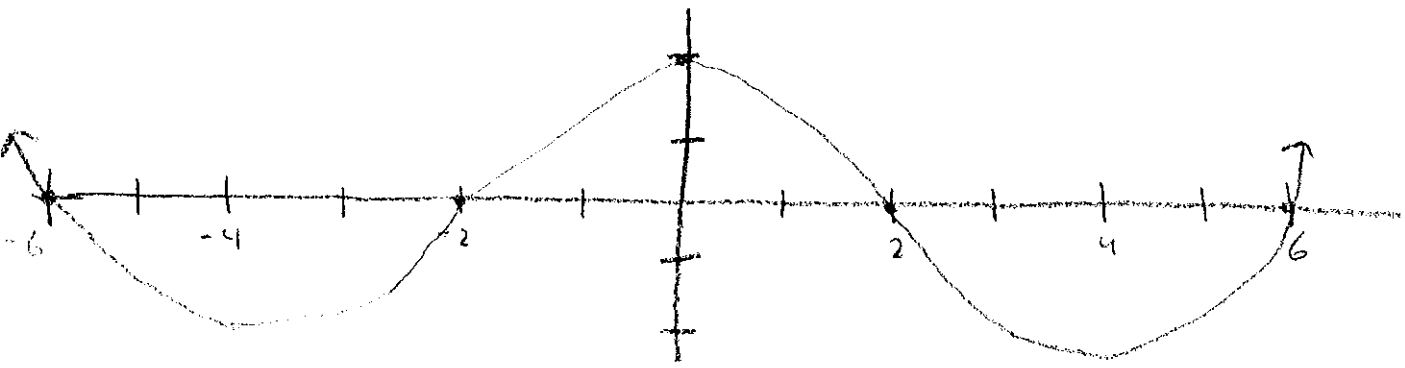
c. $2\sin(\frac{1}{2}(x+2))+8$

d. left phase shift 2, period 4π

e. left phase shift 2, period of π

f. right phase shift 2, period of 2π

3. Write out the equation for the following graph.

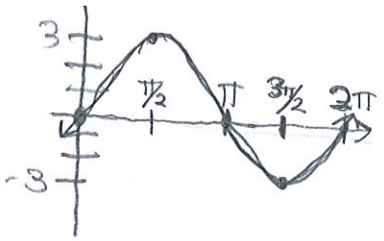


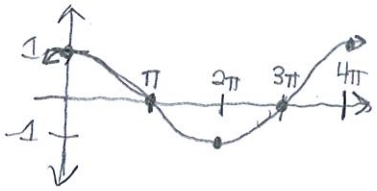
4. Graph the following equation: $\frac{1}{2}\sin(3x-3\pi)+2$

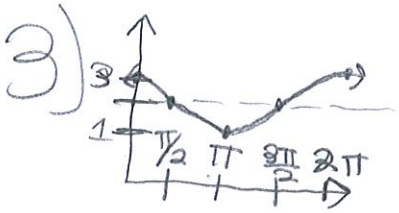
5. Briefly explain why the starting point is relevant in finding the equation of a graph. Give examples.

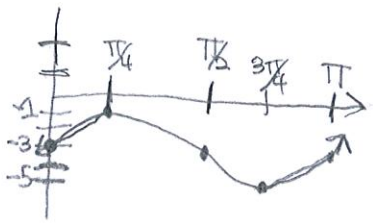
5.2A

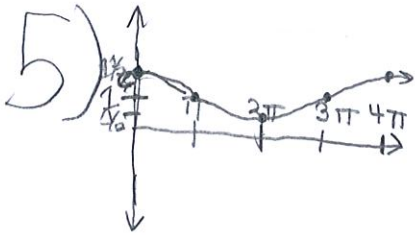
Answer Key:

1) 

2) 

3) 

4) 

5) 

5.2 B

Solutions to Practice Problems:

1.) $y = 2\sin\left(\frac{1}{2}x\right)$

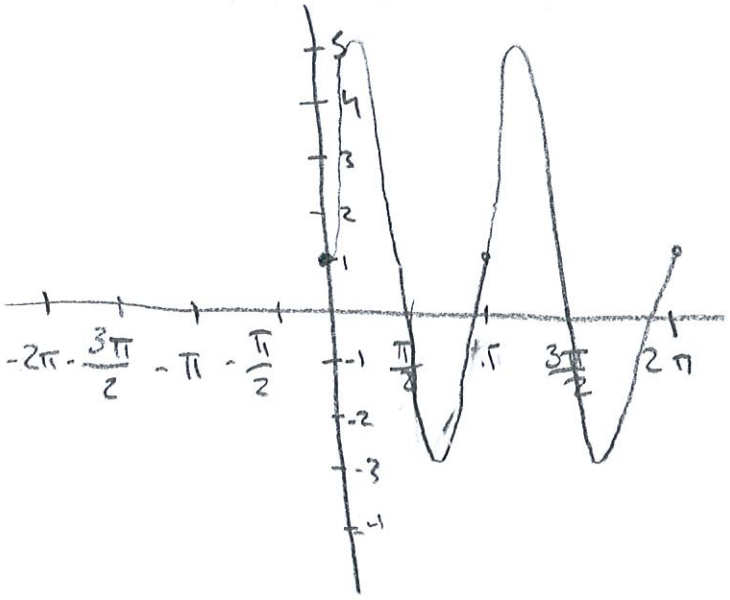
2.) $y = \cos(2x) - 1$

3.) $y = \frac{1}{2}\cos(x)$

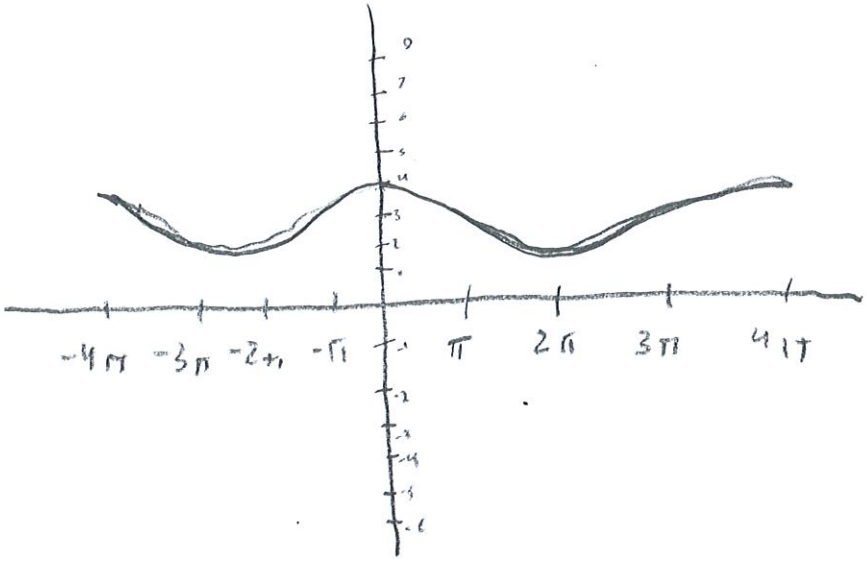
answer Key

5.2 B

4) $y = 4\sin(2x) + 1$



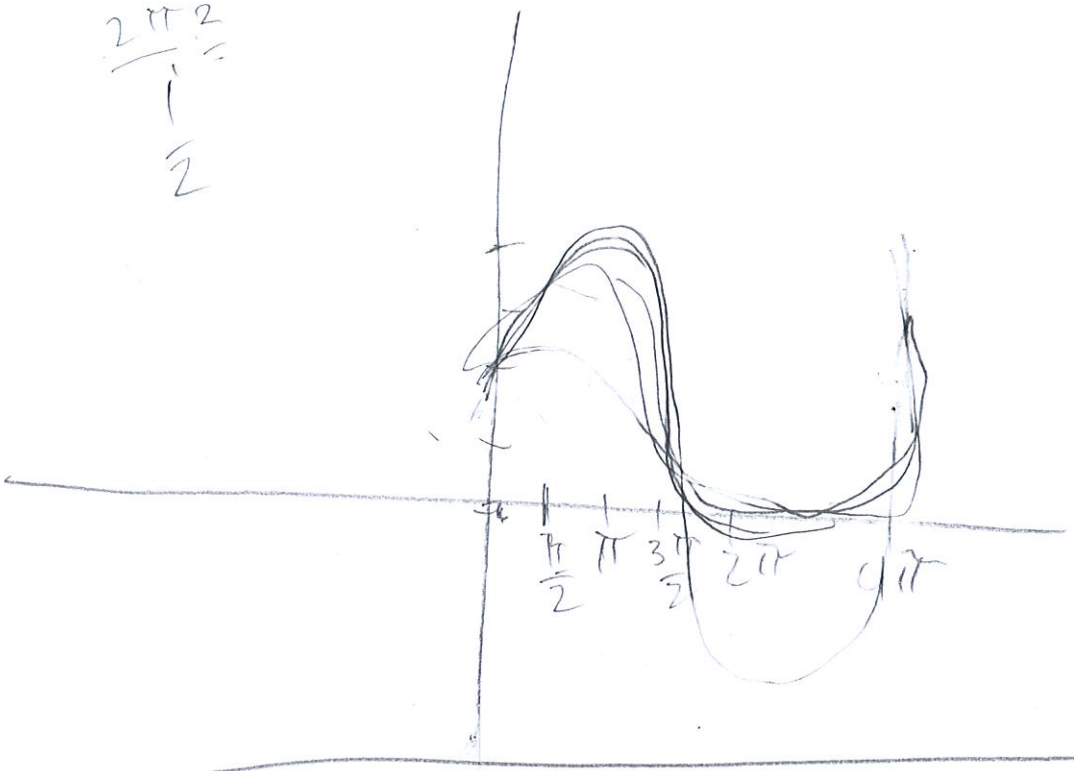
5.) $y = \cos\left(\frac{1}{2}x\right) + 3$



5.2 Key

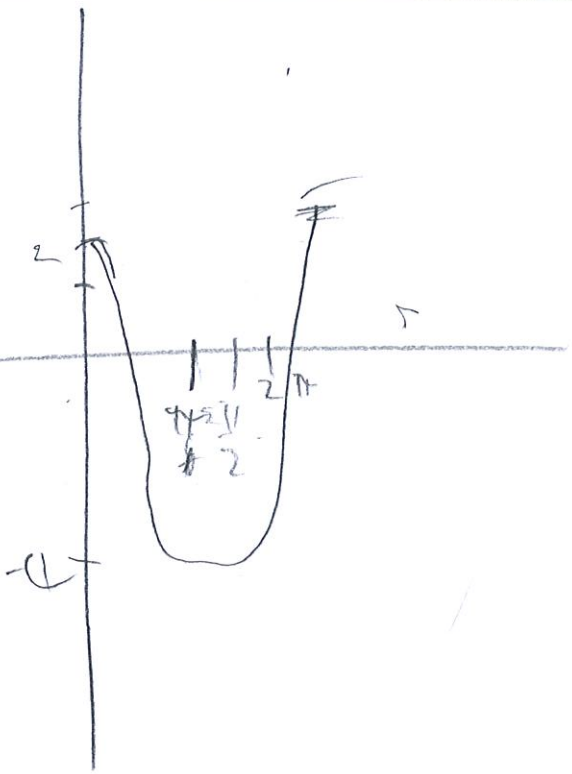
#1. $\frac{2\pi}{1} = 2\pi$

$\frac{1}{2}$



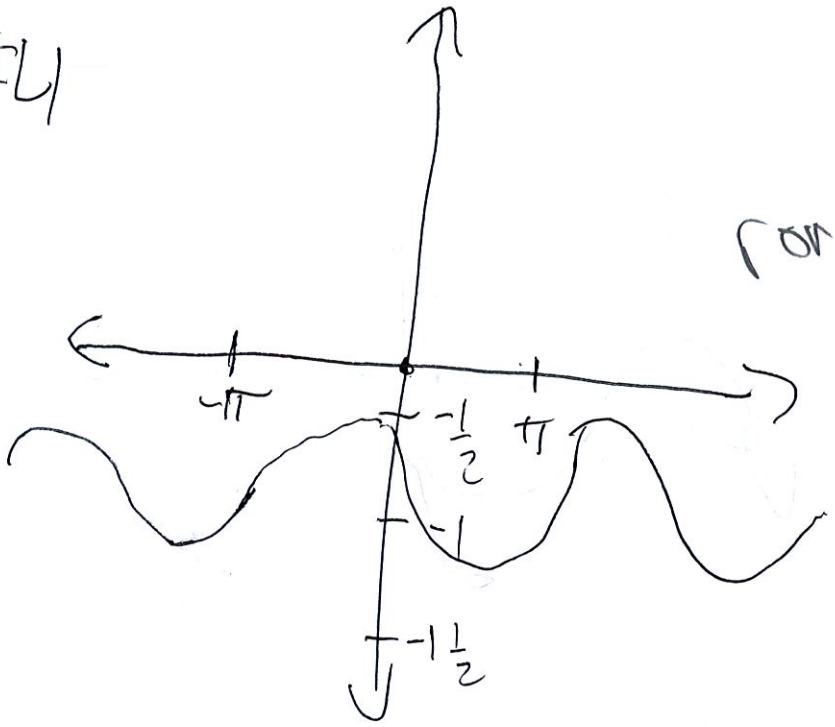
#2. $\sin(2x)$

#3a.



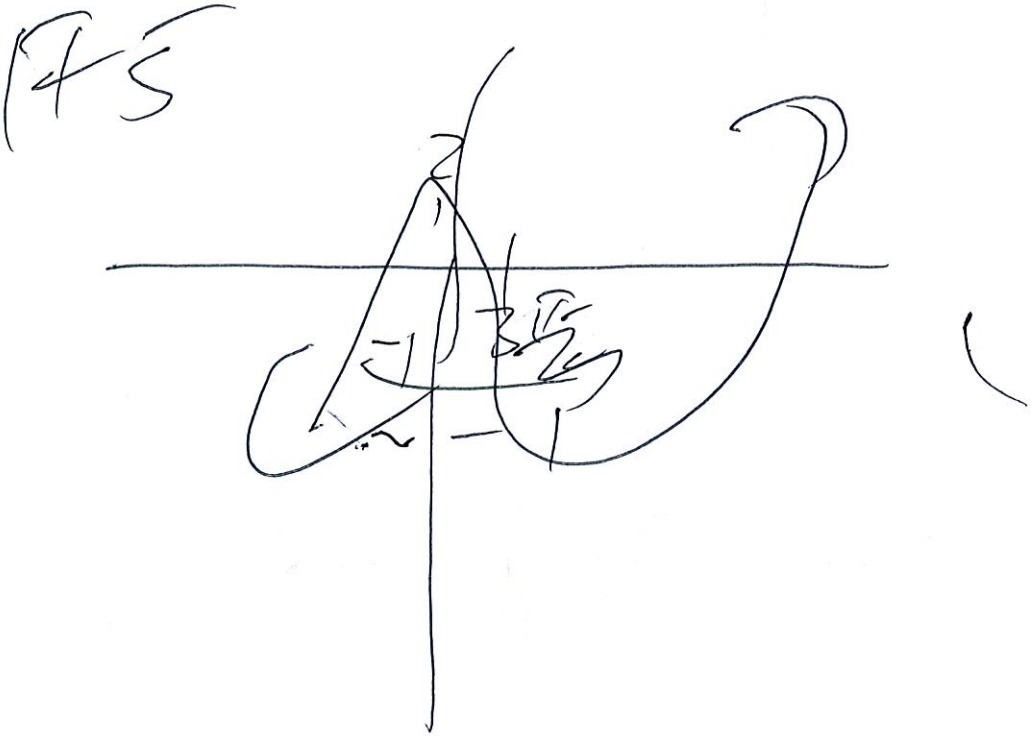
5.3A Key

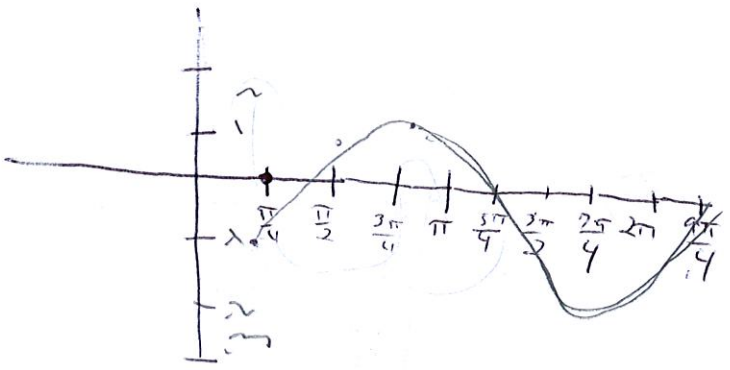
#4



rough

#5





1. a) cosine
phase shift : right $\frac{1}{2}\pi$
period : π

b) sine
period : 8π
amplitude : 2
phase shift : left 4π

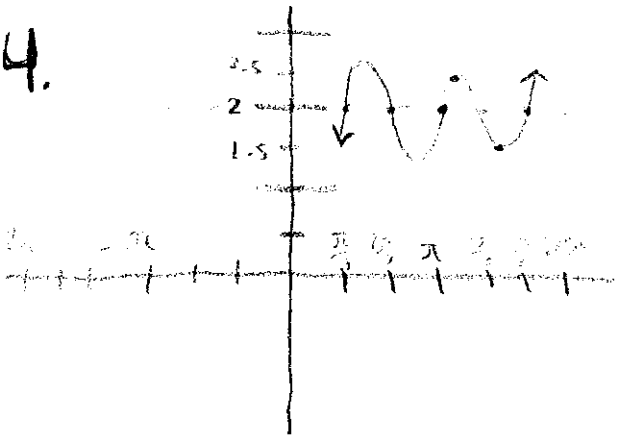
c) cosine
phase shift : left 4π
amplitude : $\frac{1}{2}$

3. $2\sin\left(\frac{\pi}{4}x + \frac{\pi}{2}\right)$

5.3c
Answer key

2. a + f
b + e
c + d

4.



5. The starting point is very important to take notice of when writing the equation of a graph because otherwise the type of function, sine or cosine, and horizontal shifts are impossible to determine. Multiple equations can therefore end up with identical graphs. For example, $\sin(x)$ graphs the same as $\cos(x - \frac{\pi}{2})$. The shift of 2π is only clear with a labelled starting point.